## Master in Mathematical Physics

## **Probability Theory**



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ABSTRACT: We begin with a study of some basic results of probability theory, including Kolmogorov's 0-1 law and limit theorems. The emphasis will be on the ideas, rather than technically complicated results. We next turn to the theory of martingales, for which we establish various inequalities and convergence results. In the final part of the course, we study discrete-time Markov processes. In particular, we deal with the questions of ergodicity and mixing, and describe a general approach due to Doeblin for proving those properties.

## Syllabus

- 1. Independence and Kolmogorov's 0-1 law
- 2. Weak and strong laws of large numbers
- 3. Cramer's theory of large deviations
- 4. Convergence of probability measures. Kantorovich-Rubinstein theorem
- 5. Central limit theorem
- 6. Conditional expectation. Discrete-time martingales
- 7. Markov chains
- 8. Ergodic theory. Doeblin's method of coupling

## References

- P. BILLINGSLEY: Probability and Measure, John Wiley, 1995.
- R. DUDLEY: *Real Analysis and Probability*, Cambridge University Press, 2002.
- T. LINDVALL: Lectures on the Coupling Method, Dover, 1992.
- D. STROOCK: Probability Theory, Cambridge University Press, 1999.