Master in Mathematical Physics

Partial Differential Equations

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ABSTRACT: The aim of this course is to provide a self-contained introduction to partial differential equations (PDEs), mainly focusing on linear equations, but also providing some perspective on nonlinear equations. The course starts with a detailed introduction to the theory of distributions, going from the basic properties of distributions to resolution of some classical PDEs. The second part is devoted to the study of the Fourier transform acting on tempered distributions. In the third part, we apply the tools developed in the first two parts to solve linear versions of some equations of the mathematical physics such as, for example, linear wave, Schrödinger, and heat equations. In the last part, we study the well-posedness of the nonlinear heat equation and some basic properties of its solutions.

SYLLABUS

1. Introduction to distributions: *function spaces, operations on distributions, order, support, first applications to linear PDEs*
2. The Fourier transform: *Schwartz space, tempered distributions, basic properties of the Fourier transform, Paley–Wiener theorem, Sobolev spaces, applications*
3. Linear partial differential equations: *linear wave, Schrödinger, and heat equations, Dirichlet problem for the Laplacian, elements of the spectral theory of the Laplacian*
4. Nonlinear heat equation: *existence and uniqueness of solution of the Cauchy problem, parabolic regularisation, dissipative properties, elements of the theory of attractors*

REFERENCES

- S. ALINHAC, P.GÉRARD: *Pseudo-differential Operators and the Nash-Moser Theorem*, AMS, 2007.
- L. C. EVANS: *Partial Differential Equations*, AMS, 2010.
- G. FOLLAND: *Introduction to Partial Differential Equations*, Princeton, 1995.
- F. G. FRIEDLANDER: *Introduction to the Theory of Distributions*, Cambridge University Press, 1999.
- R. TEMAM: *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*, Springer, 1997.