Master in Mathematical Physics

Functional Analysis

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ABSTRACT: Many problems in Mathematics lead to linear problems in infinite-dimensional spaces. In this course we shall mainly study infinite-dimensional normed linear spaces and continuous linear transformations between such spaces. We will study Banach spaces and prove the main theorems of this subject (Hahn–Banach, open mapping, uniform boundedness). The last part of the course will be devoted to bounded and unbounded operators, with specific mention of differential operators in L^2 spaces.

SYLLABUS

1. Normed vector spaces: *distance, norm, bases, Lebesgue spaces, linear maps*
2. Linear functionals: *Hahn–Banach theorem, weak convergence, Banach–Alaoglu theorem*
3. Operators in Banach spaces: *Baire Category Theorem, Open Mapping Theorem, Inverse Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Theorem*
4. Hilbert spaces: *Cauchy–Schwarz inequality, polarisation identity, Riesz Representation Theorem, orthonormal basis*
5. Operators in Hilbert spaces: *adjoint, spectrum, compact operators, unbounded operators, Hellinger–Toeplitz Theorem, closed operator, some examples*

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- E. KREYSZIG: *Introductory Functional Analysis with Applications*, John Wiley, 1989.
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- W. RUDIN: *Functional Analysis*, McGraw-Hill, 1973.