Dynamical Systems

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ABSTRACT: The main concern of the theory of Dynamical Systems is to study the evolution of a system (for example a mechanical system) with time. This includes differential equations (the time is continuous), iterations of functions (the time is discrete) and more generally any action of a group on a set. The aim of this course is to describe fundamental concepts that allow for a description of the possible asymptotic behaviors of a dynamical system. We shall thus study various notions of recurrence such as periodicity, transitivity, minimality, ergodicity... We shall also try to define what the term *chaotic behavior* means by introducing the notion(s) of entropy and hyperbolicity. An important framework will be that of Ergodic Theory.

SYLLABUS

1. Topological Dynamical Systems: *Recurrence, transitivity, minimality, topological mixing. Invariant measures, ergodicity, Poincaré recurrence theorem, ergodic theorems (Birkhoff & Von Neumann). Van der Waerden Theorem*
2. Spectral Theory: *Spectral theorem, pure point spectrum, weak mixing*
3. Examples: *In dimension 1: circle homeomorphisms, expanding maps, interval exchange transformations. The role of hyperbolicity. Shifts*
4. Entropy (metric and topological): *Shannon–McMillan–Breiman theorem. K-systems*

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